Progress in the Thermal Sciences: *AIChE Institute Lecture*

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Rather than merely surveying progress in the prediction of heat transfer over the past half century, attention is focused on the factors that have led to significant advances in understanding and practice. An almost one-to-one correspondence is demonstrated between advances in heat transfer and those in computer hardware and software. However, the development of specialized algorithms for flow and heat transfer by individual investigators appears in most cases to be an essential ingredient. Direct contributions of experimentation are relatively minor, but the synergy of combined experimentation and numerical analysis is most often the key to discovery and innovation. A quiet revolution has occurred in the form of correlative equations, and the primary and almost sole contribution of closed-form analysis has been the derivation of asymptotes for that purpose. Finally, progress in heat transfer has occurred primarily in discrete steps, and most often as a consequence of exploratory research and/or the observation and pursuit of the explanation for an anomaly.

Introduction

The first Institute Lecture was given at the Annual Meeting in Pittsburgh in 1949 by William H. McAdams and was on the subject of *Heat Transfer*. I was present at his talk, and the intervening period of 49 years corresponds closely to that of my own career in research and teaching in this very field. In that half of a century, the experimental and analytical advances in heat transfer, per se, have been rather modest but those associated with machine computation have been overwhelming, particularly in heat-exchanger design and the numerical solution of differential and integral models. The net result is a total change in the character of the thermal sciences and their role in our profession. Heat transfer has virtually disappeared as a distinct field of industrial practice and academic research in chemical engineering, but remains a subject of critical importance in traditional chemical and materials processing, as well as in every current new endeavor of chemical engineers from biomedical technology to the manufacture of computer chips. As an example of a traditional application, an endothermic homogeneous reactor may be considered to be essentially a pure heat exchanger in that the rate of reaction is proportional to the rate of heat transfer and nearly independent of the chemical kinetic mechanism. As an example at the modern end of this spectrum of applications, the speed and mechanical design of the largest computers are now limited by the rate of heat removal. For these reasons, heat transfer has rightfully retained a prominent place in the curriculum of chemical engineering.

This article describes a few important advances in the past half century that have led to the present state of the thermal sciences, but it is not intended to be a review in the conventional sense. The future is of greater interest than the past, and, as a guideline for the achievement of future advances, I will try to identify what prompted some of those of the recent past. The illustrative examples are chosen almost exclusively from the work of my students and associates, because only then are the true original objectives and the often indirect paths of discovery and innovation known to me. These examples are merely intended to be representative of the many contributions that have led over the past half century to a better understanding of heat transfer in both a qualitative and a quantitative sense. A recounting and evaluation of the even more important progress in practical applications of heat transfer is deferred to others who are better positioned to do so.

Natural Convention

The first and most lengthy set of examples herein of innovation and progress in heat transfer are taken from the field of natural convection because of the existence of a closely interconnected line of research on this process over the entire fifty years. A brief chronicle of the advances that have ensued is first presented, then an identification of the motivations for individual segments of the work, and finally a disclosure of influential aspects that went unreported.

A Chronicle

Beginning in 1953, William R. Martini attempted to take advantage of the recent acquisition by the University of Michigan of its first scientific electronic digital computer by obtaining finite-difference solutions of the unsteady-state partial-differential equations of conservation that we presumed to govern natural convection inside a long horizontal cylinder heated isothermally on one vertical half-circumference and cooled isothermally on the other. He succeeded in solving the energy balance using his own experimental values for the velocity field, but was unable to obtain stable solutions of the mass and momentum balances simultaneously with the energy balance, or even separately using his own experimental values for the temperature field (Martini and Churchill, 1960). In retrospect that latter failures were a consequence of the choice of inappropriate numerical algorithms, as well as of the limitations of the computer, an IBM650 with magnetic-drum storage that required programming in machine language and input on punched cards. This ''failure'' was, however, very instructive and may rightfully be considered to be a forerunner of all the successful solutions that have followed more or less directly.

J. David Hellums, with the advantage of a more advanced computer, an IBM704 with vacuum tubes, and a knowledge of the experiences of Martini, succeeded in obtaining unsteady-state solutions, first for the ''practice'' problem of free convection from a vertical plate whose temperature was suddenly elevated and maintained, and then for the problem studied by Martini. The transient solution for the vertical plate revealed an unexpected oscillatory approach to the previously well-known steady-state solution. The subsequent solution for the cylindrical region agreed closely with the velocity fields, temperature fields, and rates of heat transfer determined experimentally by Martini. The key to the success of these computations was the innovative use of the upwind finite-difference representation based on the velocity vector at each point of space and time. These computations are believed to constitute the first-ever complete numerical ones for a partial-differential model for either free or forced convection and, hence, the point of origin of the immense body of numerical solutions for convection that have characterized the past fifty years (Hellums and Churchill, 1961, 1962).

James O. Wilkes soon thereafter undertook unsteady-state numerical solutions for natural convection in long rectangular channels heated and cooled isothermally on the opposing vertical surfaces. The computed patterns of circulation were strongly two-dimensional (2-D) as contrasted with the quasi 1-D circulation inside a horizontal cylinder. Wilkes used a transistorized computer, an IBM7090, for this work and expressed the equations of conservation in terms of the stream function and the vorticity rather than in terms of the velocity only. He utilized centrally divided differenced in space and an implicit alternating-direction formulation in time, as well as successive over-relaxation. This formulation proved to be more efficient computationally than that of Hellums and is still widely used today. One surprising result was the initial appearance after the onset of heating and cooling of two opposing circulatory cells that eventually coalesced into one (Wilkes and Churchill, 1966).

Michael R. Samuels next utilized the same computer, but an improved programming language, Michigan Algorithmic Decoder (MAD), to obtain steady-state solutions for the still more difficult problem of natural convection in finite rectangular enclosures heated from below and cooled from above, an extension of the classical Rayleigh-Bénard problem of horizontal plates of infinite extent. He predicted a series of roll-cells of width approximately equal to their height and axes parallel to the shorter horizontal dimension of the enclosure. The drag on the ends of the roll cells was neglected in order to restrict the computations to two dimensions. In order to expedite the numerical calculations, he modified the algorithm of Wilkes by introducing a false-transient term in the stream-function equation, thereby rendering it parabolic rather than elliptic and simplifying the process of integration without affecting the steady-state solution. This innovative and highly effective procedure, which was initially proposed by Peaceman and Rachford (1955) for forced convection, has since been used by many investigators of natural convection. A plot of the numerically predicted values of the Nusselt number vs. the Rayleigh number appeared to extrapolate to unity at increasing values of the Rayleigh number as the Prandtl number was decreased, implying a dependence of the critical Rayleigh number on the Prandtl number for finite enclosures as contrasted with the well-known Prandtl-number-independent value for unbounded parallel plates. This latter implication has since been proven to be false, but much more detailed and precise computations than were feasible at that time were needed to do so (Samuels and Churchill, 1967).

Dudley A. Saville branched out from this chain of numerical solutions for natural convection in enclosures by developing a single, rapidly converging, series solution for *free* convection from a broad class of horizontal cylinders and vertically axisymmetric bodies in the thin-laminar-boundary-layer regime (Saville and Churchill, 1967, 1969, 1970).

Humbert H.-S. Chu resumed the chain of development of numerical methods and solutions for enclosures. He used essentially the same finite-difference model and procedure as those of Samuels to determine the optimal location and size of a heating panel in the vertical wall of a living or working place. By plotting the steady rate of heat transfer, as represented by the Nusselt number, vs. the rate of circulation, as represented by the maximum value of the stream function, he discovered that two widely differing rates of circulation may be achieved for the same rate of heating by the choice of the vertical location of the heater, a result of obvious physiological importance (Chu and Churchill, 1976).

All of the above solutions, both numerical and analytical, were based on the postulate of 2-D motion. 3-D solutions for natural convection were first achieved by Khalid Aziz using a formulation of George J. Hirasaki in terms of the *vector potential*, a 3-D analog of the stream function. Both worked with J. David Hellums at Rice University. (See Aziz and Hellums (1967) and Hirasaki and Hellums (1968).) Their computations were limited to a cubical enclosure, but Hiroyuki Ozoe and coworkers at Okayama University, Japan and the University of Pennsylvania extended the methodology to rectangular enclosures heated from below and then to inclined and doubly inclined ones. In steady 2-D laminar convection the pattern of flow is most effectively revealed by plots of the loci for fixed values of the stream function, which are equivalent to fluid-particle paths. However, the vector potential does not serve this role in 3-D laminar convection. Accordingly, they constructed fluid-particle paths from interpolated values of the discrete computed components of the velocity. The resulting patterns of circulation, which they subsequently confirmed experimentally, proved to be very effective in explaining the experimentally observed, radical variations in the total heat flux with the two angles of inclination, and the otherwise puzzling effects of nonuniform heating and partial baffles. Some time before software packages for this purpose became available for personal computers, they further developed an algorithm for the *dynamic* display of particle paths from any visual point-of-view on a cathode-ray tube. From an analysis of the computed particle paths, they concluded that 3-D ones inside an enclosure are necessarily concentric double-helices or degenerate forms thereof, a conclusion that may have implications even outside fluid mechanics (Ozoe et al., 1976, 1977, 1978, 1979a, b, 1983a; Chao et al., 1981, 1989; Yamamoto et al., 1982). It should be noted that use of computed particle paths for the display of the 3-D patterns of circulation in natural convection in enclosures was developed independently in the same frame of time by Mallinson and de Vahl Davis (1977).

Ozoe and coworkers also observed that the particle paths in both horizontal and singly inclined rectangular enclosures, heated and cooled on the broad sides, defined segregated roll-cells similar to those observed by Samuels. They utilized this concept to develop a generalized correlating equation for the overall rate of heat transfer in rectangular enclosures of arbitrary aspect ratios and angle of inclination (Ozoe et al., 1982).

Paul K.-B. Chao and coworkers carried out a definitive study of the extrapolation of finite-difference solutions for natural convection in enclosures to zero grid-size and discovered that most of the computed values in the literature were in significant error due to the failure to carry out such an extrapolation. Ozoe and coworkers subsequently found that most of the results obtained by prior investigators using finite-element methods suffer from the same shortcoming. They concluded that the usual statement that "the errors were reduced below (some arbitrary limit)" is simply not good enough and should not be accepted in our literature (Churchill et al., 1981; Ozoe et al., 1986).

The above-mentioned computations were all for the laminar regime. However, in the late 1970s, Shyy-Jong Lin, as well as several independent contemporary investigators, developed numerical solutions for turbulent free convection from a heated isothermal vertical plate using a κ-ϵ model. His results are in good agreement with experimental data for the entire laminar, transitional, and turbulent regimes. Ozoe and coworkers, as well as several other contemporary investigators, extended such modeling to enclosed regions. However, their complementary, precise measurements of the velocity field near a heated surface of the enclosure by laser-Doppler anemometry revealed that these measurements could not be matched in fine detail using any set of the arbitrary numerical coefficients in the κ-ϵ model. It may be inferred that despite its superficial success, the κ-ϵ model, at least in this instance, provides only a gross approximation (Lin and Churchill, 1978; Ozoe et al., 1983b).

A recent attempt to model Czochralski crystallization serves as the final example in this nearly fifty-year-long chain of progress in predicting and interpreting natural convection. That complex process of crystallization involves independent rotation of a cylindrical crystal and a semi-cylindrical crucible containing molten silicon, solidification on the non-flat end-surface of the cooled crystal, a free surface on the rest of the melt, including minisci at the wall of the crucible and at the circumference of the crystal, a decreasing liquid level and natural convection due to heating of the wall of the crucible. The experience of Vicki B. Booker in developing a numerical solution for this process using a finite-element code (FIDAP) somewhat parallels those of Martini some forty years earlier. She achieved a convergent solution for a steady-state formulation, but the computing facilities available to her in 1997, a Cray YMA-C90 and a J90 at the Pittsburgh Supercomputing Center, proved inadequate for the unsteady-state computations required to reproduce the oscillatory behavior observed in her complementary experimental work. Ozoe and coworkers have since shown by means of finite-difference computations and experimental measurements for simpler configurations that such oscillations occur in low-Prandtl-number fluids such as molten silicon in enclosures of large dimensions even in the absence of rotation and a free surface (Booker, 1997; Nakano et al., 1998; Yamanaka et al., 1998).

This array of examples is very constrained in terms of scope and perpetrators, but it is presumed to be representative of the great progress that has been made in predicting and understanding natural convection, in particular in enclosures, over the past five decades. It is also presumed to be at least indicative of the progress made in heat transfer as a whole. This progress obviously bears a close correspondence to the development of computer hardware and software, but that is not the whole story. An essential complement has been the innovative case-by-case development or adaption of numerical algorithms by the individual investigators. Furthermore, the above chronology omits the important role of particular and practical motivations, of fortuitous errors, of apparent anomalies and perhaps most of all of the synergy of experimentation, computation, and theoretical analysis. A few such factors that were important but unmentioned in the above chronology are noted in the following two subsections.

Motivations and Simplifications

The primary motivation for the research of Martini and thereby the explanation for his choice of such a relatively complex geometry and set of thermal boundary conditions was to provide guidance for the design, operation and performance of a catalytic reactor/condenser to recombine and return as liquid water the H_2 and radioactive O_2 generated in a water-boiler-type nuclear reactor and thereby avoid the danger of a chemical detonation. Even so, the geometry was greatly simplified by eliminating the inflow of gas and the outflow of water and the chemical process itself by replacing the reactive H_2 and O_2 with air and the catalytic surface with an isothermal one. The sagacious choice of simplifications that facilitate experimentation and analysis and generalization, while retaining the essential characteristics of the behavior, is a significant but often undervalued factor in exploratory research and thereby in technical progress.

The motivation for the ensuing work of Hellums, Wilkes, and Samuels was primarily to extend and exploit the pioneering step of Martini in developing a finite-difference solution for the partial differential equations representing natural convection. This intrinsic motivation prompted Hellums to revert temporarily to a simpler problem, namely an impulsively heated vertical plate. Likewise, Wilkes reverted to a more common geometry, namely a rectangular channel, for which, however, the pattern of circulation is fundamentally 2-D rather than quasi 1-D. The postulate of 2-D behavior in the modeling of Martini, Hellums, and Wilkes is justifiable in an asymptotic sense, but that of Samuels is instead an approximation since the axial orientation of the roll-cells in the shorter direction implies a finite length.

The analysis of Saville was motivated by a very practical problem, namely heat transfer by free convection from a horizontal pipe. The computations of Chu were motivated by temporary national concern in the 1970s to save energy in space heating. His postulate of two-dimensionality was conceived as a necessary approximation because of computational limitations. Aziz, Hirasaki, and Hellums were motivated by the intrinsic challenge, as well as the practical implications of 3-D natural convection. Ozoe and Chao and their coworkers immediately recognized the potentiality of such 3-D modeling for inclined and partially baffled solar collectors, which also enjoyed great, but short-lived, favor in both Japan and the U.S. in the 1970s. The motive for the modeling of Booker and of the related work of Ozoe and coworkers has already been noted.

The application of semi-empirical models for turbulent transport for the finite-difference modeling of free convection by Lin and others, and for natural convection in enclosures by Ozoe and others, was motivated more by intrinsic than practical considerations since, as contrasted with forced convection, turbulent free convection is less commonly encountered in practice than its laminar counterpart.

The studies of the effects of discretization and of extrapolation to zero grid-size or finite-element-size by Chao, Ozoe, and coworkers were motivated in part to achieve the greatest possible accuracy for their own computations, but in even greater part by the frequent assertions or implications in the literature of heat transfer that the use of finite-element formulations or even high-order-finite-difference formulations precludes the necessity of extrapolation. As noted, these self-serving and self-deluding assertions were shown to be false.

The above examples suggest that motivations, whether individual, industrial, or national, have a significant and usually positive role in determining the path of research and thereby the rate of technical progress.

Unreported Anomalies and Alternatives

The path of investigation, as reported archivally, is usually implied to follow a logical if not a linear path. Discoveries are similarly implied, if not imputed, to be due to superior insight rather than to accidental observations or serendipity. These gilded descriptions are misleading to inexperienced investigators who are thereby discouraged by the irregular and untidy path of their own work. In addition, such reports may lead even experienced investigators to ignore anomalies and thereby fail to pursue their cause or explanation. Such anomalies may have independent value and may even serve as a springboard to discovery. Early in my career, the experimental phase of the research of Roy L. Gealer was terminated in midcourse by an accidental detonation in the process of loading that destroyed most of the equipment. The reviews of an article submitted to the *AIChE Journal* on the computational predictions and on the experiments completed before the accident were generally favorable, but asked that all references to the accident be deleted since it was unplanned and nondefinitive. In addition, the Editor (Harding Bliss) questioned whether or not detonation was within the scope of chemical engineering and the *AIChE Journal*. We convinced him that it was and persuaded him to accept one sentence concerning the accident and its probable cause. That sentence provoked far greater interest and correspondence than the balance of the article, because such accidental detonations were then not uncommon in the chemical industry. For such reasons, a few unreported but possibly important aspects of the work chronicled above are noted below.

As mentioned, the stimulus of the work of Martini was the accidental destruction of a water-boiler-type nuclear reactor by detonation of the H_2 and O_2 that was generated. This motivation, which is barely mentioned in the archival article describing the work, was the determining factor in the choice of geometry and boundary conditions. Another important, but unreported, aspect of that work was the role of G. K. Batchelor, who was with us as a Visiting Professor at the time of the planning of the research. He predicted the steady, slow, solid-body rotation of the central core of fluid inside a thin boundary layer extending around the entire circumference, and proposed that, rather than undertake imprecise experimentation and risky finite-difference modeling, Martini model this behavior analytically, as a simple extension of Batchelor's recent modeling of natural convection in a tall rectangular channel heated and cooled on the vertical surfaces. Fortunately, Martini rejected this seemingly authoritative advice politely and persisted with his own predilections on the basis of their greater challenge. He observed from his very first measurements of the velocity and temperature fields that the central core was oscillatory, but nonrotating, and that the boundary layer was of significant and highly varying thickness. These observations forced him to utilize an unsteady-state finite-difference formulation and guided his choice of a nonuniform grid spacing. Even beyond this qualitative observation and the quantitative characterization of the behavior, his measurements proved to be invaluable as a test of the validity and accuracy of the finite-difference computations of Hellums, as well as those of Martini. A fortuitous dividend of the observation of oscillations inside the cylinder by Martini, and thereby the development of an unsteady-state formulation by Hellums, was the latter's use of a reduced formulation for the test problem of free convection from a vertical plate suddenly raised to and maintained at a higher temperature and the consequent discovery that the approach to a steady state is oscillatory rather than monotonic in all geometries.

An anomaly in the work or Hellums stimulated a development of perhaps equal and certainly broader consequences than his development of a successful algorithm for the numerical solution of a partial-differential model for natural convection and its successful application. One dimensionless velocity, namely $u/(gD)^{1/2}$, was found to lead to one less dimensionless parameter in the partial-differential model than another equally valid one, namely, uD/ν. The search for an explanation for this anomaly led to the discovery of a generalized method of scaling that identifies a minimal set of dimensionless groupings for any algebraic, integral, and/or differential model for any physical and/or chemical process and thereby identifies any possible similarity transform, that is, reduction in the number of independent variables and, in turn, a reduction of a partial differential equation to an ordinary one. The functional effect of idealization and of extreme values of the parameters and variables is also readily explained by means of this procedure (Hellums and Churchill, 1961, 1962).

The computational work of Hellums and Wilkes provoked several outcries that may now be difficult to comprehend. First, several analysts expressed doubt as to the validity of such solutions and searched for some hidden or disguised source of empiricism. Second, many analysts castigated the numerical procedure and results as ugly and an unfair threat to their domain. Third, many practitioners were understandably skeptical and distrustful of results obtained by a procedure that had not even been mentioned in their formal education. This is an example of the often repeated experience that strongly disputed innovations are sometimes accepted as common wisdom only a few years later.

The computational formulations and therefore results of Samuels were influenced by some unreported visualizations of the pattern of circulation with cigar smoke. The first visualizations revealed a single 2-D roll-cell with its axis in the long horizontal dimension. After some corresponding numerical solutions were carried out, this mode of circulation was discovered to be an artifact of the slight heating of one side wall by the beam of light used to illuminate the smoke. The stable mode of circulation in the absence of such radiative heating of the side wall was found to consist of a row of quasi 2-D roll-cells with their axes in the shorter horizontal direction and with a width approximately equal to their height. A computational anomaly, as well as this experimental anomaly, was encountered but unreported by Samuels, namely the existence of multiple stationary states. Any number of cells of approximately equal widths could be established simply by the choice of the initial conditions. On the other hand, multiple stationary states were not observed experimentally, only the one corresponding to widths approximately equal to their height. This absence of multiplicity in the physical world is attributable to the unavoidable presences of one or more sources of asymmetry, such as that due to the nonlinear variation of the viscosity of the fluid with temperature or to some intentional or accidental perturbation. An unreported aspect of the work of Saville was his initial intent and attempt to carry out finite-difference solutions for free convection from a horizontal cylinder. He soon realized that a steady-state solution does not exist for a cylinder of infinite length (as implied by the postulate of two-dimensionality) in the absence of surrounding surfaces or a finite forced flow. Instead of abandoning the problem, he derived the previously mentioned analytical solution which by virtue of its simplicity, generality and rapid convergence is a far greater accomplishment than would have been the intended finite-difference solutions. The existence of the analytical solutions is a fortuitous cousequence of the idealized boundary conditions for thin-laminar-boundary-layer theory.

The reviewers of the manuscript describing the work of Chu were more sophisticated than those at the time of Hellums, Wilkes and Samuels. They accepted the concept, validity and usefulness of finite-difference solutions for natural convection but they strongly criticized the use of a nonconservative formulation. Indeed, his integrated heat flux from the heated surface failed to agree exactly with that to the cooled surface for finite grid-spacings. However, these two heat fluxes were found to provide upper and lower bounds, respectively, thereby reducing the number of grid points and in turn the total amount of computation required to extrapolate confidently to zero grid-size (Chu and Churchill, 1977). The lesson here is that a nominally erroneous formulation, just as with the false-transient model by Samuels, may be expeditious as long as its consequences and limitations are recognized.

The availability of a number of widely differing formulations and detailed solutions for use in the studies on extrapolation by Chao and coworkers was a fortuitous consequence of the posing of the problem originally solved by Wilkes as a ''benchmark'' competition by Jones and de Vahl Davis (1983).

Booker had essentially completed her original objective of developing a finite-element solution for Czochralski crystallization when her experimental measurements of the velocity field revealed that the process was actually oscillatory and thereby her steady-state solution invalid physically. Had she not begun with the false premise of the existence of a steady state, she would presumably have made no real progress computationally and might even have abandoned the investigation for a more tractable one before undertaking the experimental work.

These unreported anomalies and redirections of work illustrate a frequent and often crucial aspect of exploratory research not only in convection, but also in all phases of heat transfer and indeed in most fields of science and engineering.

Forced Convection

Most of the fundamental problems of laminar forced convection have been solved analytically in the dim past or numerically in the past half century. These numerical calculations are generally more routine than those for natural convection and, hence, will not be reviewed herein. Instead, attention is directed to the turbulent regime and, in the interest of brevity, to the geometry and condition of greatest practical importance, namely fully developed convection in a round tube, heated, or cooled externally. The same factors that were identified above in connection with natural convection might be expected to be operative in turbulent forced convection. They are operative, but the idiosyncracies associated with turbulence result in a number of distinctive characteristics. Although the work of my associates again proves useful in this respect, it is insufficient in scope to serve as a complete guideline as was more or less the case in natural convection. Of course, the motivations, anomalies and sudden revelations that inspired the work of my nonassociates are difficult to identify with confidence.

As contrasted with natural convection, the transport of momentum is uncoupled from the transport of energy, insofar as variations of physical properties with temperature may be neglected. Hence, flow is examined first and then convection.

Early Contributions to the Prediction of Turbulent Flow

A brief review of early progress in the prediction of turbulent flow is essential in order to put the advances of the last half century in perspective.

Perhaps, the greatest contribution of all time in the prediction of turbulent flow, at least in a practical sense, was the concept of space-averaging by Reynolds (1895) and his implementation of this concept to simplify the general partial differential equations of conservation to a much more tractable form. Time-averaging, which is essentially equivalent to space-averaging, has been adopted by most subsequent investigators as more convenient. Even earlier, Boussinesq (1877) proposed that the transport of momentum by the turbulent fluctuations be modeled by simply adding an effective or *eddy* viscosity to the molecular one. Prandtl (1925) subsequently proposed, by analogy to the mean-free-path of the molecules in a gas, a mixing-length model for turbulent transport. Most subsequent work on the prediction of turbulent flow and convection up to the present has been based on one or the other of these two heuristic models. Prandtl and his associates and contemporaries also derived empirical algebraic expressions for the prediction of these two quantities and used speculative dimensional analysis to deduce invaluable asymptotic expressions for the time-mean velocity distribution and thereby for the shear stress due to turbulence, the eddy viscosity, and the mixing length. von Kármań (1930) derived a semi-empirical differential expression for the mixing length that may be considered to be the precursor of the more sophistocated models that are described below. When asked by the present author about the basis for this model, he replied that it was chosen as the simplest dimensionally correct expression involving only derivatives of the time-mean velocity.

Contributions to the Prediction of Turbulent Flow during the Past Half Century

The models of Prandtl and von Kármán, with minor modifications, served as the basis for the prediction of the eddy viscosity and the mixing length until Launder and Spalding (1972) proposed calculation of the kinetic energy of turbulence κ and the rate of dissipation of turbulence ϵ from moments of the differential momentum balance, and in turn the eddy viscosity or mixing length from an expression of Kolmogorov (1941). This model and other closely related ones have been utilized extensively for predictions, but considerable approximation and empiricism is required to make them sufficiently explicit and tractable computationally.

Perhaps the greatest advance of all time in turbulent modeling other than time-averaging is that of *direct numerical simulation* (DNS), as pioneered by Orszag and Kells (1980) and others, in which the unsimplified and unaveraged equations of conservation are solved directly. This process has proven feasible because of the advent of supercomputers, but perhaps more critically because of the discovery that the grid-size needs to be reduced only to the point at which the turbulent shear is negligible relative to the viscous shear, not down to the scale of the smallest eddies, which persist to molecular dimensions. The bad news is that because of the computational demands in general, and in particular because of the decreased grid-size required by an increase in the rate of flow, the implementation of this methodology is still limited after 20 years to simplistic geometries and to rates of flow barely above the minimum for fully developed turbulence ($Re = 4000$ for a round tube).

Large eddy simulation (LES), which was devised by Schumann (1975) and others, utilizes DNS with a relatively gross grid-spacing for the largest eddies and and the κ-ϵ model or its equivalent with a finer *subgrid* for the smaller eddies. Of course, this subgrid-modeling invokes the shortcomings of the κ-ϵ and eddy-viscosity or mixing-length models insofar as it is utilized in the regions where they fail.

In view of this recent rapid, if incomplete, progress in the numerical modeling of turbulence, further significant advances are to be anticipated in the near future. Whether such advances will consist of marginal improvements in the above methodologies or follow some completely new path is, however, uncertain.

Early Contributions to the Prediction of Turbulent Convection

Because of the similarity of the time-averaged equations of conservation for momentum and energy, the early predictive expressions for turbulent convection consisted primarily of analogies in which the mechanisms for the transport of momentum and energy by the turbulent fluctuations were postulated to be identical, resulting in a great simplification. Reynolds (1874) thereby eliminated the mechanism of turbulent transport itself, and, by neglecting molecular transport completely, obtained a result that may be written in modern terminology as

$$Nu = RePr(f/2) \tag{1}$$

Prandtl (1910) improved upon the *Reynolds analogy* by including the added resistance due to pure molecular diffusion across a boundary layer of thickness δ, thereby obtaining

$$Nu = \frac{RePr(f/2)}{1+\left(\frac{f}{2}\right)^{1/2}(Pr-1)\delta^+} \tag{2}$$

The *Prandtl analogy* implies that the dependence of Nu and Re varies with Pr and that the Reynolds analogy is valid only for $Pr = 1$.

Colburn (1933) noted the numerical and functional similarity of a purely empirical correlating equation for $f/2$ with several for $Nu/RePr^n$, where n varied from 0.3 to 0.4, and thereby proposed the expression

$$Nu = Re\left(\frac{f}{2}\right)Pr^{1/3} \tag{3}$$

with the exponent of 1/3 being simply a convenient compromise between 0.3 and 0.4.

The *Colburn analogy* is correlative rather than mechanistically based and is inferior to the Prandtl analogy functionally in every respect except for the fortuitously correct asymptotic dependence on Pr for $Pr \to \infty$. It would not be mentioned here if the absence of a numerical coefficient did not erroneously imply a theoretical origin and if it were not still widely used in practice.

Contributions of the Past Half-Century to Turbulent Convection

Three significant advances in predicting turbulent convection emerged at approximately the beginning of this period. First, different rates of turbulent transport were postulated for momentum and energy as accounted for by the *turbulent Prandtl number* $Pr_t = \nu_t/\alpha_t = c\mu_t/k_t$, resulting in improved numerical predictions overall but particularly for low values of Pr. Second, molecular transport in the turbulent core was taken into account leading to greatly improved predictions for low-Prandtl-number fluids such as liquid metals. Third, turbulent transport in the viscous sublayer was taken into account leading to improved predictions for large-Prandtl-number fluids and justifying the exponent of 1/3 in the Colburn analogy in that limited sense.

Only one of the many subsequent improved analogies will be cited here, namely that of Reichardt (1951). He eliminated dy between the differential time-averaged momentum and energy balances in terms of μ_t and k_t, postulated independence of Pr_t from y and hence from $u\{y\}$, made several simplifications on the basis of the postulated relative magnitude of various terms of the combined differential balance in various regions of the cross-section, and introduced one empiricism, namely an expression for the temperature profile near the wall for very large Pr. As a consequence of these steps, he was able to integrate analytically to obtain an algebraic expression for Nu as a function of Re and Pr/Pr_t. (This expression will be examined subsequently.)

An alternative approach is exemplified by the work of Sleicher and Tribus (1957), who developed a Graetz-type series-solution for developing, as well as fully developed, convection in fully developed flow in a round tube with any arbitrary mode of heating at the wall. Because of the limitations of digital computers at that time, they chose to evaluate the eigencoefficients and eigenvalues of this solution for a wide range of values of Re and Pr by an exceedingly tedious process of trial and error using an analog computer—a now almost extinct species. Their solutions for the Nusselt number are exact in structure, but the numerical accuracy of the predictions is limited by the uncertainty of the empirical expressions they used for the time-mean velocity distribution, the eddy viscosity, and the turbulent Prandtl number, as well as by the incomplete convergence of the computations. Notter and Sleicher (1972) subsequently used a digital computer, an improved mathematical formulation, and somewhat improved empirical expressions for u, μ_t and Pr_t to update the eigenvalues and eigencoefficients for uniform heating and a uniform wall-temperature. These latter results have served as the standard in most respects until very recently.

New Model-Free Formulation for Turbulent Flow

Book writing is not ordinarily a path to discovery. However, in the process of writing a textbook on turbulent flows as a companion to those on inertial and viscous flows I was struck by the illogical and unnecessarily complex structure of much of the literature on this subject, as well as by the consequences of a century of heuristic and empirical models leavened theoretically only by asymptotic concepts and, even then, only incompletely. This perception prompted me to develop exact integral formulations for the time-averaged velocity distribution and the time-averaged and space-averaged velocity in terms of the dimensionless time-averaged turbulent shear stress, and then to develop a correlating equation with a firm theoretical structure and a minimum of empiricism for the latter quantity. At this same point of time, I became aware of the successful implementation of DNS by Kim et al., (1987), Lyons et al. (1991), and Rutledge and Sleicher (1993) to obtain essentially exact solutions for fully developed flow between unbounded parallel plates. I had been aware of the concept of DNS by Orszag and Kells (1980) and had mentioned it in the book in passing, but the appearance of the numerical results now called for extensive revisions. I temporarily put aside the completion of the book, even though it had already been copy edited, in order to assess the role and potential of DNS and my own model-free formulation, and to incorporate this new material throughout. That assessment and the pursuit of the consequences of the new developments (see Churchill, 1997) has so far been more challenging and inviting than the completion of the book.

The starting point for the new model-free formulation for turbulent flow, as well as for the heuristic ones, is the following time-averaged, once-integrated momentum balance for the steady 1-D or quasi 1-D flow of a fluid with invariant physical properties

$$r = \mu \frac{du}{dy} - \rho\overline{u'v'} \qquad (4)$$

When using the eddy-viscosity and mixing-length models, the turbulent shear stress $-\rho\overline{u'v'}$ is replaced by $\mu_t(du/dy)$ and $\rho l^2(du/dy)^2$, respectively. However, both μ_t and ℓ vary strongly with distance from the wall and with the rate of flow, and empirical expressions or secondary models are required for their prediction.

For fully developed flow in a round tube, to which attention herein will be primarily confined, $\tau = (1-(y/a))\,\tau_w$, and Eq. 4 may be reexpressed in the following dimensionless form

$$1 - \frac{y^+}{a^+} = \frac{du^+}{dy^+} + \left(\overline{u'v'}\right)^+ \qquad (5)$$

where $y^+ = y(\tau_w\rho)^{1/2}/\mu$, $a^+ = a(\tau_w/\rho)^{1/2}/\mu = Re(f/8)^{1/2}$, $u^+ = u(\rho/\tau_w)^{1/2}$ and $(\overline{u'v'})^+ = -\rho\overline{u'v'}/\tau_w$. The new formulations were initially developed starting from Eq. 5. However, the following modified representation

$$\left(1 - \frac{y^+}{a^+}\right)\left[1 - \left(\overline{u'v'}\right)^{++}\right] = \frac{du^+}{dy^+} \qquad (6)$$

where $(\overline{u'v'})^{++} = -\rho\overline{u'v'}/\tau$ was found to be superior in several respects. This quantity has well-defined physical significance as the fraction of the total shear stress due to the turbulence and must thereby be greater than zero and less than unity at all locations within the fluid.

Formal integration of Eq. 6 and reexpression in terms of $R = 1 - (y/a)$ for simplicity results in

$$u^+ = \frac{a^+}{2}\int_{R^2}^{1}\left[1-\left(\overline{u'v'}\right)^{++}\right]dR^2 = \frac{a^+}{2}(1-R^2) - \frac{a^+}{2}\int_{R^2}^{1}\left(\overline{u'v'}\right)^{++}dR^2 \quad (7)$$

while integration over the cross-section by parts then leads to

$$\left(\frac{f}{2}\right)^{1/2} = u_m^+ = \frac{a^+}{4}\int_0^1\left[1-\left(\overline{u'v'}\right)^{++}\right]dR^4 = \frac{a^+}{4} - \frac{a^+}{4}\int_0^1\left(\overline{u'v'}\right)^{++}dR^4 \quad (8)$$

A particular merit of expression of the turbulent shear stress in terms of $(\overline{u'v'})^{++}$ was the resulting recognition of the possibility of reduction of the double integral for u_m^+ to the single one of Eq. 8 by virtue of integration by parts. This latter simplification is possible in terms of $(\overline{u'v'})^{+}$, μ_t/μ and ℓ/a, but was not generally recognized because of the more complex resulting forms.

Equations 7 and 8 are exact but an empirical correlating equation is required for $(\overline{u'v'})^{++}\{y^+, a^+\}$. For this purpose, Churchill (2000) has proposed the following

$$\left(\overline{u'v'}\right)^{++} = \left(\left[0.7\left(\frac{y^+}{10}\right)^3\right]^{-8/7} + \left|\exp\left\{\frac{-1}{0.436y^+}\right\} - \frac{1}{0.436a^+}\left(1+\frac{6.95y^+}{a^+}\right)\right|^{-8/7}\right)^{-7/8} \quad (9)$$

The individual terms of Eq. 9 correspond to the theoretical asymptotic structures for $y^+ \to 0$, $30 < y^+ < 0.1a^+$ and $y^+ \to a^+$, respectively. The coefficient of 7×10^{-4} is based on the DNS of Rutledge and Sleicher (1993) for a parallel-plate channel, the coefficients 0.436 and 6.95 are based on the recent, precise experimental measurements of the time-averaged velocity distribution in a round tube by Zagarola (1996), and the exponent of $-8/7$ is based on the experimental measurements of Wei and Willmarth (1980) for parallel-plate channels. The postulated interchangeability of the expressions and values of $(\overline{u'v'})^{++}$ and u^+ for a round tube in terms of a^+ and for a parallel−plate channel in terms of b^+ is based on the analogy of MacLeod (1951). The combinations of Eqs. 7 and 9 and of Eqs. 8 and 9 are presumed to produce more accurate predictions of $u^+\{y^+, a^+\}$ and $u_m^+\{a^+\}$, respectively, for $a^+ > 300$ than any other predictive formulations or correlations in the literature. The limitation to $a^+ > 300$ is based on the experimental disappearance of the intermediate regime of "overlap" as represented by the exponential term of Eq. 6. Equations 7 and 8 reveal that the contribution of the turbulence is simply a deduction from the expressions for purely laminar flow at the same value of a^+, a result that is not so readily evident from the classical formulations in terms of the eddy viscosity and the mixing length.

Reassessment of Prior Models for Turbulent Flow

Comparison of prior models for the prediction of turbulent flow with the new model-free formulations is very revealing. For example, eliminating du^+/dy^+ between Eq. 6 and the equivalent expression in terms of μ_t/μ indicates that

$$\frac{\mu_t}{\mu} = \frac{\left(\overline{u'v'}\right)^{++}}{1-\left(\overline{u'v'}\right)^{++}} \quad (10)$$

from which it follows that the eddy viscosity is independent of its heuristic diffusional origin and furthermore is well-behaved (finite and bounded) for all values of y^+. On the other hand, application of this same procedure for ℓ/a results in

$$(\ell^+)^2 = \frac{\left(\overline{u'v'}\right)^{++}}{\left(1-\frac{y^+}{a^+}\right)\left[1-\left(\overline{u'v'}\right)^{++}\right]^2} \quad (11)$$

from which it follows that the mixing length is also independence of its heuristic origin as an effective mean-free-path, but is unbounded at the centerline. Equivalent results and conclusions follow for a parallel-plate channel, but for all other 1-D channels, including circular concentric annuli, μ_t/μ and $(\ell^+)^2$ are found from such a comparison to be unbounded at one location and negative over an adjacent finite region although $(\overline{u'v'})^{++}$ remains well-behaved. It is difficult to explain why these failures have been overlooked for so long and so universally since they are quite apparent when even the older relevant sets of experimental data are reexamined critically.

The various κ-ϵ models, all of which function by predicting the eddy viscosity or the mixing length, share these failures. The LES models share these failures insofar as the singularities in the eddy viscosity and mixing length occur in the region in which the subgrid model is applied. The κ-ϵ models, and thereby the LES subgrid model, actually incorporate a greater degree of approximation and empiricism than does Eq. 9, but the net effects are difficult to compare quantitatively because of their presence in differential and algebraic equations, respectively. The μ_t/μ, ℓ/a, $\kappa - \epsilon$ and LES models may produce results of fair accuracy, even for geometries in which they are fundamentally inapplicable, because their failures occur only in particular regions and because of, rather than in spite of, their empirical constants.

Model-Free Formulations for Turbulent Convection

The thermal analogs of the dimensional and dimensionless forms of the momentum balances are

$$j = -k\frac{dT}{dy} + \rho c\overline{T'v'} \quad (12)$$

and

$$\frac{j}{j_w}\left[1-\left(\overline{T'v'}\right)^{++}\right]=\frac{dT^+}{dy^+} \tag{13}$$

where $T^+=k(\tau_w\rho)^{1/2}(T_w-T)/\mu j_w$ and $(\overline{T'v'})^{++}=\rho c\overline{T'v'}/j$, the local fraction of the transport of energy by the turbulent fluctuations. However, despite the superficial similarity of Eqs. 6 and 13, the thermal behavior is much more complex. It is convenient to rewrite Eq. 13 as

$$(1+\gamma)\left(1-\frac{y^+}{a^+}\right)\left[1-\left(\overline{u'v'}\right)^{++}\right]\left(\frac{Pr_T}{Pr}\right)=\frac{dT^+}{dy^+} \tag{14}$$

where, by definition, $1+\gamma=(j/j_w)/(1-(y^+/a^+))$ and Pr_T/Pr is to be interpreted as simply a symbol for $(1-(\overline{T'v'})^{++})/(1-(\overline{u'v'})^{++})$. The quantity γ represents the magnitude of the deviation of the heat flux density distribution from the shear stress distributions across the channel, an effect that has often been overlooked or ignored. Equation 14 may be integrated formally and reexpressed in terms of R to obtain

$$T^+=\frac{a^+}{2}\int_{R^2}^{1}(1+\gamma)\left[1-\left(\overline{u'v'}\right)^{++}\right]\left(\frac{Pr_T}{Pr}\right)dR^2 \tag{15}$$

For uniform heating,

$$1+\gamma=\frac{1}{R^2}\int_0^{R^2}\left(\frac{u^+}{u_m^+}\right)dR^2 \tag{16}$$

which, by virtue of Eqs. 4 and 5, and integration by parts, may be expressed in terms of $(\overline{u'v'})^{++}$ as follows

$$1+\gamma=\frac{\left(\frac{1-R^2}{R^2}\right)\int_0^{R^2}\left[1-\left(\overline{u'v'}\right)^{++}\right]dR^4+\int_{R^4}^{1}\left(\frac{1-R^2}{R^2}\right)\left[1-\left(\overline{u'v'}\right)^{++}\right]dR^4}{\int_0^1\left[1-\left(\overline{u'v'}\right)^{++}\right]dR^4} \tag{17}$$

It follows from Eqs. 15 and 16 and again by means of integration by parts that

$$Nu=\frac{2a^+}{T_m^+}=\frac{2a^+}{\int_0^1 T^+\left(\frac{u^+}{u_m^+}\right)dR^2}$$

$$=\frac{8}{\int_0^1(1+\gamma)^2\left[1-\left(\overline{u'v'}\right)^{++}\right]\left(\frac{Pr_T}{Pr}\right)dR^2} \tag{18}$$

The quantity γ, which is small with respect to unity, may readily be evaluated numerically using $(\overline{u'v'})^{++}$ from Eq. 6. Somewhat more convenient formulations than Eqs. 16 and 18, insofar as numerical evaluations and reduced expressions for special cases are concerned, result from replacing Pr_T by $Pr_t=((\overline{u'v'})^{++}/(\overline{T'v'})^{++}\ \ ((1-(\overline{T'v'}))^{++}/(1-(u'v')^{++})$ to obtain

$$T^+=\frac{a^+}{2}\int_{R^2}^{1}\frac{(1+\gamma)\,dR^2}{1+\frac{Pr}{Pr_t}\left(\frac{\left(\overline{u'v'}\right)^{++}}{1-\left(\overline{u'v'}\right)^{++}}\right)} \tag{19}$$

and

$$Nu_D=8\Big/\int_0^1\frac{(1+\gamma)^2\,dR^4}{1+\frac{Pr}{Pr_t}\left(\frac{\left(\overline{u'v'}\right)^{++}}{1-\left(\overline{u'v'}\right)^{++}}\right)} \tag{20}$$

It may be recognized from the definitions of Pr_T and Pr_t that

$$\frac{1}{Pr_T}=\frac{\left(\overline{u'v'}\right)^{++}}{Pr_t}+\frac{\left(1-\left(\overline{u'v'}\right)^{++}\right)}{Pr} \tag{21}$$

The primary advantage of Eqs. 19 and 20 is that Pr_t is better behaved than Pr_T. For example, for $Pr=0$, Eq. 20 reduces to

$$Nu_0=8\Big/\int_0^1(1+\gamma)^2\,dR^4=\frac{8}{(1+\gamma)^2_{mR^4}} \tag{22}$$

Equation 22, which predicts the lower bounding value of Nu for any given value of Re or a^+, is invaluable in identifying outliers in plots of experimental data, as well as serving as an asymptotic component of a correlating equation for all Re and Pr. Also, for $Pr\rightarrow\infty$, the complete thermal development occurs very near the wall, where from Eq. 9 $(\overline{u'v'})^{++}\cong 0.7(y^+/10)^3$. Then, insofar as Pr_t approaches a fixed value Pr_t^* at the wall, Eq. 15 may be integrated analytically to obtain the equivalent of

$$Nu_\infty=0.07343\left(1-\frac{Pr_t^*}{Pr}\right)^{4/3}\left(\frac{Pr}{Pr_t^*}\right)^{1/3}Re\left(\frac{f}{2}\right)^{1/2}$$

$$\cong 0.07343\left(\frac{Pr}{Pr_t^*}\right)^{1/3}Re\left(\frac{f}{2}\right)^{1/2} \tag{23}$$

Equations 22 and 23 follow from Eq. 18 and 15, respectively, only in conjunction with Eq. 21. Insofar as $Pr_t=Pr$ for all y^+ for some particular value of Pr, Eq. 20 reduces to

$$Nu_1=8\Big/\int_0^1(1+\gamma)^2\left(1-\left(\overline{u'v'}\right)^{++}\right)dR^4 \tag{24}$$

which, by virtue of Eq. 8, may be reexpressed as

$$Nu_1=Re\left(\frac{f}{2}\right)\Big/(1+\gamma)^2_{wmR^4} \tag{25}$$

where wmR^4 implies the integrated-mean value, weighted by $1-(\overline{u'v'})^{++}$. A particular merit of Eqs. 22, 23 and 25 is that they are independent, at least explicitly, of the highly uncertain values and variation of Pr_t.

For a uniform wall-temperature, Eq. 16 is replaced by

$$1+\gamma = \frac{1}{R^2}\int_0^{R^2} \frac{u^+}{u_m^+}\left(\frac{T^+}{T_m^+}\right) dR^2 \tag{26}$$

Equations 15, 19, and 23 remain applicable, but not Eqs. 16–18, 20, 22, 24 and 25. Although Eqs. 19 and 26 may be solved iteratively for γ, T^+ and T_m^+, their differential form proves to be more convenient computationally (Yu et al., 2000). The particular solution for $Pr = 0$, as determined from Eq. 19, is

$$Nu_0 = 4\left(T_{co}^+/T_{mo}^+\right)/\left(1+\gamma_{mR^2}\right) \tag{27}$$

and that for $Pr_t = Pr$

$$Nu_1 = 4\left(T_{co}^+/T_{mo}^+\right)\left(u_m^+/u_c^+\right)/\left(1+\gamma_{wmR^2}\right) \cong Re(f/2)/\left(1+\gamma_{wmR^2}\right) \tag{28}$$

Equation 23 remains applicable for Nu_∞

Before considering the determination of Nu for general values of Pr, the quantity Pr_t will be examined.

Turbulent Prandtl Number

The turbulent Prandtl number Pr_t was originally introduced on the presumption that its variation would be more constrained than that of the eddy conductivity. This has proven to be the case. The turbulent Prandtl number has often been scorned by theorists because of its heuristic diffusional origin, but its expression in terms of $(\overline{u'v'})^{++}$ and $(\overline{T'v'})^{++}$ as a consequence of the model-free relationships of the previous section would appear to dismiss the basis for such misgivings. On the other hand, the detailed numerical and functional behavior of the turbulent Prandtl number is still not well-defined, particularly near the wall and for small values of the molecular Prandtl number. A few of the most relevant contributions to the prediction of this important quantity during the past half century, and particularly in the last few years, will be mentioned here.

Peter H. Abbrecht, inspired by the aforementioned work of his fellow graduate student, Charles A. Sleicher, made improved measurements of the time-mean velocity distribution, particularly very near the wall, and unique 2-D measurements of the time-mean temperature distribution following a discrete step in wall temperature. He concluded (Abbrecht and Churchill, 1960) from the 2-D temperature gradients that the turbulent Prandtl number was independent of the temperature field and equal to that in a parallel-plate channel for the same values of a^+ and b^+. The first of these conclusions was greeted with two vociferous and contradictory responses—one that such a result was to be expected and hence not worthy of mention, and the other that such results and their interpretation were obviously in error. The second conclusion was simply ignored. Remarkably, neither conclusion appears to have been proven or disproven or even retested experimentally in the ensuing 40 years. However, both conclusions have gained implicit acceptance, and the turbulent Prandtl is now generally, but not always, postulated to be the same function of μ_t/μ and hence of $(\overline{u'v'})^{++}$, as well as of Pr in all 1-D flows.

Reynolds (1975) reviewed predictive methods and expressions for Pr_t and concluded that none were completely satisfactory, particularly near the wall, perhaps because of a dependence on y^+ and a^+, as well as on μ_t/μ and Pr. Two primary advances have since occurred. Direct numerical simulations do not require the specification of Pr_t and in principle could be used to predict it. In practice, such predictions have so far been limited to small Reynolds numbers and, with one exception, to values of Pr of the order of unity. Papavassiliou and Hanratty (1997) used both Eulerian and Lagrangian direct numerical simulations to predict Pr_t for a wide range of Pr. Their predictions for $Pr > 100$ indicate an unbounded increase as $y^+ \rightarrow 0$, which contradicts the premise of the derivation of Eq. 23, but suggests that it may still be a satisfactory approximation for lesser but still large values of Pr. Yahkot et al. (1987) used a *renormalization group* (RNG) method to derive an implicit algebraic expression for Pr_T as a function of μ_T/μ and Pr, and Elperin et al. (1996) have since improved upon the single independent exponent and coefficient therein. These latter expressions appear to be in qualitative, but not exactly quantitative, accord with experimental data, particularly near the wall. Kays (1994) recently reviewed the experimental data and predictive expressions for Pr_t and concluded that the RNG methodology may be applicable only to the region of overlap (the semilogarithmic regime of the velocity distribution). He proposed an empirical expression for Pr_t than may be rewritten in terms of $(\overline{u'v'})^{++}$, rather than μ_t/μ, as follows

$$Pr_t = \frac{2\left[1-\left(\overline{u'v'}\right)^{++}\right]}{Pr\left(\overline{u'v'}\right)^{++}} + 0.85 \tag{29}$$

He notes that Eq. 29 predicts an unbounded value of Pr_t at the wall for all values of Pr and suggests that a value of unity rather than this expression be used near the wall.

The development of a comprehensive predictive or correlative expression for the turbulent Prandtl number is the principal remaining challenge with respect to the prediction of turbulent forced convection.

Numerical Predictions

Equation 20, together with an empirical expression similar to Eq. 29, was used by Heng et al. (1998) to predict values of Nu for a uniformly round tube for a wide range of values of Re and Pr. Danov et al. (2000) carried out similar calculations for parallel-plate channels heated uniformly and equally or heated isothermally on one plate and cooled isothermally on the other. Yu et al. (2000) have carried out corresponding calculations for both a uniformly heated and an isothermal round tube. These three sets of values, which will be examined subsequently in terms of correlation, are presumed to be the most accurate in the literature owing to the use of Eq. 9 rather than separate, incongruous, and relatively less accurate values for u^+ and μ_t/μ. However, the results are subject to some small numerical improvement when a more reliable expression is devised for the prediction of Pr_t.

Correlation

An examination of the subject of correlation logically follows those on natural convection and turbulent forced convection since the raw results of numerical computations as well as of experimental measurements are ordinarily in the form of tabulations of discrete values.

Early Correlations

The state of the art in the correlation of experimental data at the time of McAdams' 1949 Institute Lecture is readily discerned from the second (1942) edition of his then-definitive book "Heat Transmission". For example, the data for forced convection were generally represented by logarithmic plots of Nu vs. Re and Pr and then in turn algebraically by expressions such as

$$Nu = ARe^n Pr^m \tag{30}$$

with the coefficient and exponents determined from straight-line fits. The characteristically wide scatter in such correlations has usually been attributed explicitly or implicitly to experimental error. However, this explanation is not applicable for numerically computed values, which are ordinarily not subject to scatter and are by definition free of secondary effects. Nusselt (1909) was apparently the first to apply dimensional analysis to the set of variables presumed to define turbulent forced convection in a channel. In so doing, he determined the characteristic grouping that justifiably bears his name, but he unjustifiably postulated that h was proportional to a power of each of the other variables and thereby obtained Eq. 29 instead of the correct expression, namely

$$Nu = \varphi\{Re, Pr\} \tag{31}$$

This mistake has plagued our profession beyond any measure, and to this day is the source of much misdesign and overdesign.

New Generalized Form of Correlation

In 1972, I undertook to correlate presumably exact numerically computed values of Nu for free convection from an isothermal vertical plate in the thin-laminar-boundary-layer regime. It seemed appropriate to incorporate the following known, exact asymptotes

$$Nu = 0.6004 Gr^{1/4} Pr^{1/2} \quad \text{for} \quad Pr \to 0 \tag{32}$$

and

$$Nu = 0.5027 Gr^{1/4} Pr^{1/4} \quad \text{for} \quad Pr \to \infty \tag{33}$$

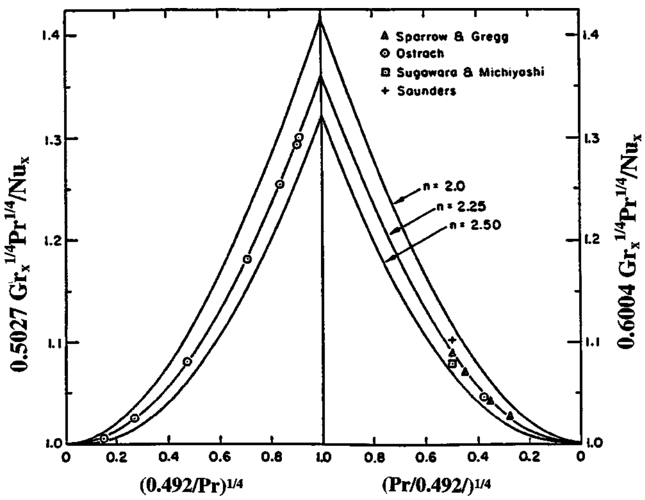

Figure 1. Representation by Eq. 35 of experimental and computed values for the local heat-transfer coefficient for laminar free convection from an isothermal vertical plate (Churchill and Usagi, 1972).

rather than determine some mean value for the exponent of Pr as implied by expressions of the fixed-power type such as Eq. 30. It occurred to me that one such possible expression was

$$Nu^n = (0.6004 Gr^{1/4} Pr^{1/2})^n + (0.5027 Gr^{1/4} Pr^{1/4})^n \tag{34}$$

which may be reexpressed more simply as

$$\frac{Nu}{0.5027 Gr^{1/4} Pr^{1/4}} = \left[1 + \left(\frac{Pr}{0.492}\right)^{n/4}\right]^{1/n} \tag{35}$$

Equation 35 with $n = -9/4$ is seen in Figure 1 (from Churchill and Usagi, 1972) to represent both computed and experimental values almost exactly. Somewhat inexplicably, the same exponential value of $-9/4$ has proven to a satisfactory choice for free convection with uniform as well as isothermal heating, from bodies of all shapes, and even for natural convection within enclosures.

Equation 35 may be generalized for all types of convection as

$$\frac{Nu}{Nu_\infty} = \left[1 + \left(\frac{Nu_0}{Nu_\infty}\right)^n\right]^{1/n} \tag{36}$$

where Nu_∞ and Nu_0 represent asymptotic expressions for large and small values of Pr, Gr, Re, and so on, and indeed for almost all phenomena, including fluid-mechanical, thermodynamic, rheological, economic and physiological behavior, as

$$\frac{y\{x\}}{y_\infty\{x\}} = \left[1 + \left(\frac{y_0\{x\}}{y_\infty\{x\}}\right)^n\right]^{1/n} \tag{37}$$

where $y_\infty\{x\}$ and $y_0\{x\}$ represent asymptotic expressions for large and small values of x. Correlations with the form of Eq. 37 are now rapidly replacing logarithmic plots and algebraic fixed-power expressions in the literature of heat transfer because of their greater accuracy, generality, and convenience. In retrospect, it is evident that most of the scatter in the graphical correlations of the past is due to unrequited parametric effects and varying power-dependences rather than to experimental error.

Some caution must be exercised in applying Eq. 37. For example, the asymptotes $y_0\{x\}$ and $y_\infty\{x\}$ must be free of singularities, must intersect once and only once, must both be upper and lower bounds, and preferably both provide the same degree of approximation for near-asymptotic values.

We were not the first to use power-mean combinations of asymptotes for correlation. Our contribution has been to generalize such usage, to explore its sequential applications, to identify the requirements for the asymptotes, and to propose standardized procedures for determination of a rounded-off value for the arbitrary exponent(s). It is interesting to note that Eq. 37 has the form of the famous Diophantine problem that provided the basis for *Fermat's Last Theorem*.

Equation 37 may be applied sequentially for three regimes starting with $y_0\{x\}$ as follows

$$y\{x\} = \left(\left[(y_0\{x\})^n + (y_i\{x\})^n\right]^{m/n} + \left[y_\infty\{x\}\right]^m\right)^{1/m} \quad (38)$$

The reverse order of combination is equally valid, but in general the same values will not be obtained for n and m, resulting in two expressions of equal justifiability, but perhaps different success, in terms of representation of the data. This process may be extended to any number of regimes and also applied sequentially or by nesting for two or more independent dimensionless variables.

Churchill (1977) applied this sequential procedure for fully developed forced convection in a round tube with either a uniformly heated or an isothermal wall with five regimes and two independent variables to obtain

$$Nu^{10} = Nu_\ell^{10} + \left[\frac{e^{(2{,}200 - Re)/365}}{Nu_\ell^2} + \left(Nu_0 + \frac{0.079\,Re\left(\frac{f}{2}\right)^{1/2} Pr^{1/3}}{(1 + Pr^{-4/5})^{5/6}}\right)^{-2}\right]^{-5} \quad (39)$$

Here Nu_ℓ designates the theoretical value for laminar flow (3.657 for an isothermal wall and 48/11 for a uniform heat flux density), the exponential term is an arbitrary representation for the transitional regime, Nu_0 represents the limiting value for $Pr = 0$ in the turbulent regime ($\cong$ 4.8 for an isothermal wall and $\cong$ 6.3 for uniform heating), and the term in $Pr^{1/3}$ the asymptote for $Pr \to \infty$. The term in $Pr^{-4/5}$ is based on the erroneous postulate of $Nu \; \alpha \; RePr(f/2)^{1/2}$ for $Pr \to 0$ in the turbulent regime. The four arbitrary exponents, two of which were chosen to be unity, are based on the computed values of Notter and Sleicher (1972) and culled experimental data for both heat and mass transfer. The following, corresponding and complementary expression was proposed for the friction factor

$$\left(\frac{f}{2}\right)^{12} = \left(\frac{8}{Re}\right)^{12} + \left[\left(\frac{37{,}530}{Re}\right)^{16} + \left|2.46\ln\left\{\frac{1}{1.13\,Re(f/2)^{1/2}}\right\}\right|^{16}\right]^{-3/2} \quad (40)$$

Equation 40 encompasses three regimes of flow and incorporates two arbitrary exponents. The success of Eqs. 39 with f from 40 is demonstrated in Figure 2.

In some applications Eq. 38 incorporates a fundamental flaw. In the typical case, in which $y_0\{x\}$ is a lower bound and $y_i\{x\}$ an expression for the transitional regime that falls below $y_0\{x\}$ for very small x and above $y_\infty\{x\}$ for very large x, it reduces for $x \to 0$ to

$$y\{x\} = \frac{y_0\{x\}}{\left[1 + \left(\frac{y_0\{x\}}{y_\infty\{x\}}\right)^p\right]^{1/p}} \quad (41)$$

where $p = -m > 0$. Equation 41 is incongruous since $y_0\{x\}$ is a lower bound. This dicrepancy may be tolerable numerically if $y_0\{x\} \ll y_\infty\{x\}$ and/or $p \gg 1$. The analogous discrepancy actually occurs in both Eqs. 39 and 40 for $Re \to \infty$, but is negligible for all practical purposes. This discrepancy may be avoided completely by interpreting $y^n - y_0^n$ as the dependent variable in the second combination, thereby obtaining

$$y^n - y_0^n = \left(y_i^{nm} + \left[y_\infty^n - y_0^n\right]^m\right)^{1/m} \quad (42)$$

In Eq. 42 and hereafter, the functional notation is omitted in the interests of simplicity and clarity but a possible dependence on x is implied for y, y_0, y_∞ and y_i. For the previously mentioned "typical" case, Eq. 42 may be reexpressed as

$$\frac{y^n - y_0^n}{y_\infty^n - y_0^n} = \frac{1}{\left[1 + \left(\frac{y_\infty^n - y_0^n}{y_i^n}\right)^m\right]^{1/m}} \quad (43)$$

It is obvious that for positive values of n and m, $y_0 \le y \le y_\infty$, thereby exorcizing the possible anomaly of Eq. 38. Equation 42 has a more complex structure than Eq. 38, but incorporates exactly the same number of functions and arbitrary exponents. An alternate expression of equal merit to Eq. 42 may be formulated by the reverse order of combination.

One common problem with both Eqs. 39 and 42 and their complementary reverse-order combinations is that very few sets of experimental data for $y\{x\}$ have sufficient precision and scope to support the determination of unique expressions for y_0, y_i, and y_∞ and unique values for n and m. In

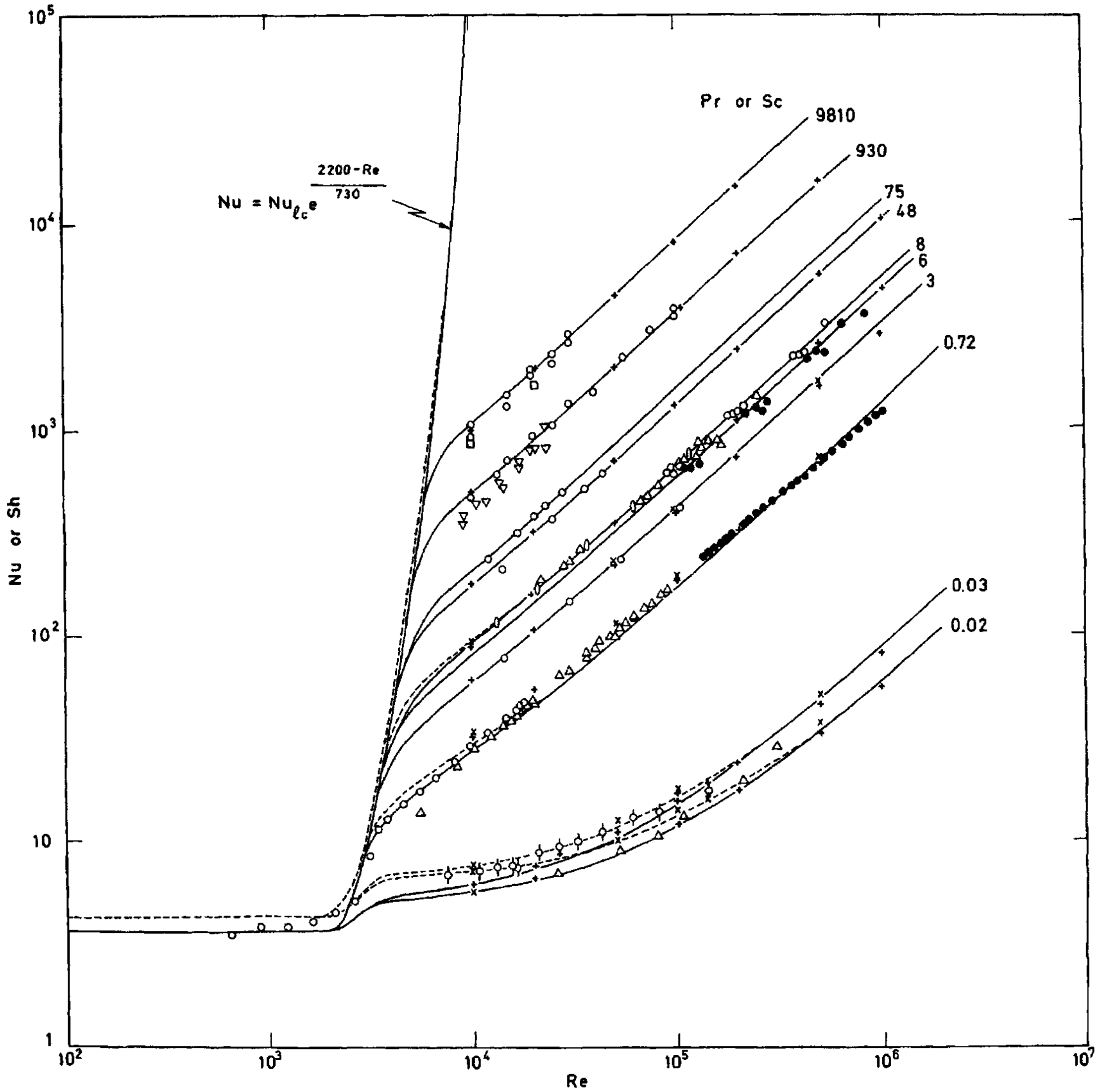

Figure 2. Representation by Eq. 39 of computed and culled experimental data for heat and mass transfer in forced convection in a round tube.

– – –Uniformly heated; ——isothermal. Reprinted with permission from Churchill (1977), Copyright (1977) American Chemical Society.

many instances the function y_0 and y_∞ may be known in advance, as for example from Eqs. 22 and 23, thereby reducing this difficulty somewhat. In some instances, setting $n = -m = \pm 1$ as an approximation and taking up the slack in y_i may result in an acceptable representation.

Improved Correlating Equations For Turbulent Convection

Churchill et al. (2000) recognized that the previously mentioned analogy of Reichardt, which was derived for a uniform wall-temperature, could be interpreted to have the form

$$Nu = \frac{1}{\left(\frac{Pr_t}{Pr}\right)\frac{1}{Nu_1} + \left(1 - \frac{Pr_t}{Pr}\right)\frac{1}{Nu_\infty}} \qquad (44)$$

even though the expression Reichardt obtained for Nu_∞ was seriously in error. The proper expression for Nu_∞ is that given by Eq. 23, while that for Nu_1, which was correct in the original, is given by Eq. 28. Churchill et al. further discovered that Eq. 44, when rearranged as

$$\frac{Nu - Nu_1}{Nu_\infty - Nu_1} = \frac{1}{1 + \left(\frac{Pr_t}{Pr - Pr_t}\right)\frac{Nu_\infty}{Nu_1}} \qquad (45)$$

has the same structure as Eq. 43 with $n = -m = 1$, and therefore that

$$Nu_i = \left(\frac{Pr - Pr_t}{Pr_t}\right)\left(\frac{Nu_\infty - Nu_1}{Nu_1}\right)Nu_\infty \qquad (46)$$

Equations 45 and 46 constitute a remarkable result in several respects. First, the dependence of Nu on Pr is seen to be transitional in the classical sense from a lower bound, up through a point of inflection, to an upper bound. Such complex functional behavior has never been recognized experimentally or predicted theoretically for this extensively studied system. Second, the effective independent variable $(Pr - Pr_t)/Pr$ is not, as might have been expected, a simple power of Pr. Third, this transformation stretches the range of the independent variable down to zero at $Pr = Pr_t$ and allows Nu_1 to be interpreted as an asymptotic function rather than as a particular intermediate value.

The applicability of Eq. 44, and thereby Eq. 45, is obviously limited to $Pr \geq Pr_t$, a consequence of one of the simplifications made by Reichardt in the derivation of his analogy. Churchill et al. (2000) devised the following complementary expression for $Pr \leq Pr_t$

$$\frac{Nu - Nu_0}{Nu_1 - Nu_0} = \frac{1}{1 + \left(\frac{Pr_t - Pr}{Pr}\right)\left(\frac{Nu_\infty^1 - Nu_1}{Nu_1 - Nu_0}\right)\left(\frac{Nu_1}{Nu_\infty^1}\right)} \quad (47)$$

For a uniform wall-temperature Nu_1 is again given by Eq. 28, Nu_0 is given by Eq. 27, and $Nu_\infty^1 \equiv Nu_\infty\{Pr_t = Pr\}$. The latter term, which appears in Eq. 47 by virtue of matching of the derivatives of Nu with respect to Pr/Pr_t at $Pr_t = Pr$, is equal to 0.07343 $Re(f/2)^{1/2}$. The characteristics of Eq. 47 are similar to those of Eq. 45, namely the presence of a stretched variable rather than a power of Pr and prediction of a transition from a lower bound to an upper bound with a point of inflection.

Equations 45 and 47 are both free from any explicit empiricism and are independent of the expressions used to predict Pr_t, Nu_0, Nu_1 and Nu_∞, although, of course, the numerical predictions of Nu are not. Equations 45 and 47 are not exact; some error is present in Eq. 44 and thereby in Eq. 45 due to the simplifications made by Reichardt to obtain an analytical solution, principally the postulate of independence of Pr_t from y^+. Equation 47 has very little theoretical justification; it is merely conjectural.

Because of the generalized form of Eq. 44, Churchill et al. (2000) further conjectured that it, and thereby Eqs. 45 and 47, might be directly applicable for other 1-D channels and other modes of heating. The success of Eqs. 45 and 47 in predicting the computed values of Yu et al. (2000) for a round tube with a uniform well-temperature is demonstrated in Figure 3 and the equivalent success in predicting the computed values of Heng et al. (1998) for a uniformly heated round tube and those of Danov et al. (2000) for a parallel-plate channel with both uniform equal heating and unequal uniform wall-temperatures in Figure 4. These comparisons do not constitute a test of the dependence of Pr_t on Pr since the same expression was used in both the computations and the correlating equations.

It is concluded that Eqs. 45 and 47 constitute a significant step forward in the prediction of turbulent convection in channels. Not only are they numerically successful for a broad range of conditions, as illustrated in Figures 3 and 4, but they predict a complexity of functional behavior that has never before been recognized. For example, they imply that Nu is not a fixed power-function of either Re or Pr over any extended range, which totally contradicts Eq. 30.

As a final note, Eq. 39 may continue to serve as a predictive expression for all regimes of flow and both uniform and isothermal heating if Eqs. 45 and 47 are simply utilized in place of $Nu_0 + 0.079 Re(f/2)^{1/2} Pr^{1/3}[1 + Pr^{-4/5}]^{-5/6}$. Tabu-

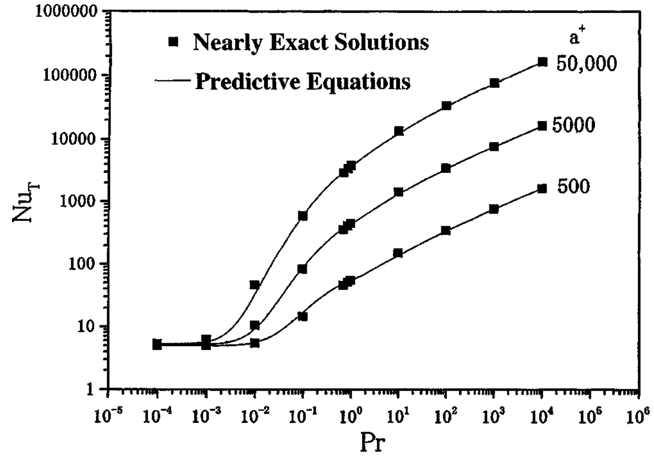

Figure 3. Representation by Eqs. 45 and 47 of computed values for turbulent forced convection in an isothermally heated round tube (Yu et al., 2000).

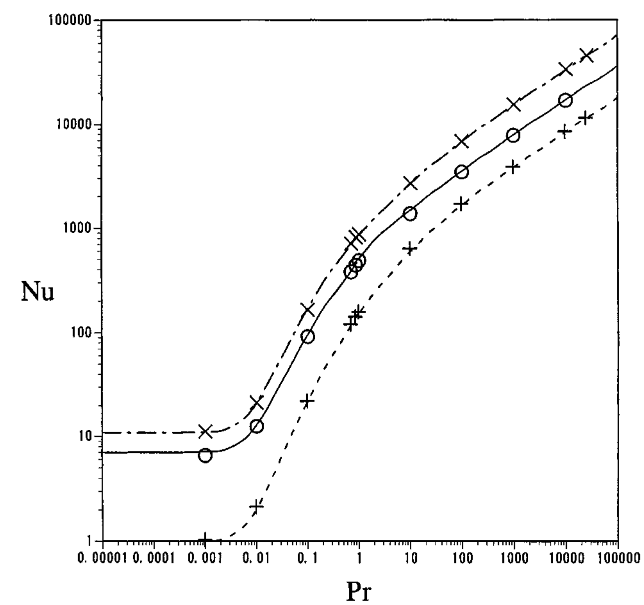

Figure 4. Representation by Eqs. 45 and 47 of computed values for turbulent forced convection in uniformly heated channels (Churchill et al. 2000).

x: Uniformly and Equally Heated Parallel Plates, $b^+ = 5000$; O: Uniformly Heated Round Tubes, $a^+ = 5{,}000$; + Parallel Plates, One Uniformly Heated and the Other Uniformly and Equally Cooled, $b^+ = 5{,}000$.

lated values, as well as empirical correlating equations for Nu_0 and Nu_1, are given by Heng et al., Danov et al., and Yu et al. Replacement of the term within the absolute value signs of Eq. 40 by

$$3.2 - \left(\frac{554}{Ref^{1/2}}\right) + \left(\frac{100}{Ref^{1/2}}\right)^2 + \frac{1}{0.436}\ln\left\{\frac{Re(f/8)^{1/2}}{1 + 0.301\left(\frac{e}{a}\right)Re\left(\frac{f}{8}\right)^{1/2}}\right\} \quad (48)$$

provides suitable values of f for use with Eqs. 45 and 47, as well as the presumably best prediction in the literature for the friction factor itself for $Re(f/2)^{1/2} > 600$.

The development of correlating equations is seen from the above to be a creative art in itself. Correlation is worthy of the attention of all investigators since their experimental and/or computed values may survive in the long run only in that form.

Diverse Examples of Progress in Heat Transfer

Several singular examples that illustrate the interaction of practical motivations, experimental observations, and theoretical analysis in discovery, innovation, and progress in diverse aspects of heat transfer will conclude this survey.

Thermal Radiation

In the early 1950s, Professor C. M. Sliepcevich invited me to collaborate with him in a civil-defense-inspired investigation of the effectiveness of natural and artificial clouds in blocking thermal radiation from a nuclear weapon set off in the atmosphere. One day, a visiting project officer asked me what would happen to the cloud itself. I replied off-handedly, ‘‘almost nothing’’. Apparently, this answer was considered to be too absurd or flippant to merit refutation since it was received with stony silence. However, about a year later he phoned from the Pentagon to say ‘‘How did you know?’’ The long delay was apparently due to the wait for a cloudy day in Nevada in order to carry out a full-scale test. I knew the answer in advance of the question because (Sleicher and Churchill, 1956) had at my behest already investigated that process theoretically and found that the small fraction of the incident thermal flux absorbed by each tiny droplet of water in a cloud is almost instantaneously transferred by conduction to the much greater mass of the immediately surrounding air until such time as the temperature of the air as a whole begins to rise significantly. This prediction and field test also explain the persistance of clouds and fogs despite the radiative thermal flux from the sun.

Bert K. Larkin utilized the experimental and analytical skills gained in that investigation of radiative transfer through the atmosphere to study the contribution of thermal radiation to heat transfer through fibrous and foamed insulations. In the furtherance of his investigation, he derived the first complete analytical solution of the Maxwell equations for the interaction of electromagnetic radiation and a single cylinder (Larkin and Churchill, 1959a). His numerical evaluations of the infinite series of complex functions comprising this solution implied the existence of an optimal fiber-diameter for which the net radiative transfer through the insulation would be a minimum. However, no such minimum was observed in his experiments with real insulations. Eventually, he speculated that a seemingly small, but finite, distribution of fiber size in the test insulations of nominally fixed-fiber size might obscure the minimum. Subsequent computations for a size distribution and experiments with insulations made up of monofibers both confirmed his speculation—a result of obvious practical importance (Larkin and Churchill, 1959b). Lawrence B. Evans extended the solution of the Maxwell equations to encompass hollow cylinders and found almost the same blockage for the resulting greatly reduced mass of fibers (Evans et al., 1964). John C. Chen extended the experimental concepts and predictive models to characterize radiative transfer at high temperatures in packed beds (see Chen and Churchill, 1963) and with his coworkers at Lehigh University to fluidized beds (Cimini and Chen, 1987). The potential benefit of extending the experience gained in one practical application to another, as well as the synergetic effect of anomalous observations and theoretical analyses, are well illustrated by this brief set of examples in the field of thermal radiation.

Performance of a Double-Spiral Heat Exchanger

The thermal behavior of a double-spiral heat exchanger with a rectangular cross-section and reversed flow was recently investigated by Mark R. Strenger and Matthew J. Targett in connection with its proposed use by W. B. Retallick as a catalytic incinerator for cigarette smoke, microorganisms, and other contaminants of air in working and living spaces, and in particular in hospitals and airplanes (Strenger et al., 1990). In this application, the purpose of the exchanger is to heat air to above 600°C with only a small net input of energy at the core and thereby to the surroundings. The numerical solution of a model for that process predicted a maximum increase of temperature for an intermediate rate of flow, although one had not been observed in our experimental work up to that time or, for that matter, by any prior investigators. By means of analytical solutions of a somewhat idealized model (Targett et al., 1992), the maximum was shown to be a consequence of the complex thermal coupling of the entering stream with the two adjacent exiting streams and vice versa. Forewarned by these analyses, a maximum increase in temperature was confirmed experimentally by a better choice of operating conditions. The failure of prior investigators to identify this important aspect of behavior was in part due to their choice of conditions, but primarily because of their unfortuitous choice of the *thermal effectiveness*, which has a very flat maximum, or the *correction factor for the log-mean temperature-difference*, which has none, rather than the dimensionless temperature rise as a criterion of performance.

The first series of experimental devices failed to produce a sufficient temperature rise. The numerical model again came to the rescue by demonstrating the extreme sensitivity of the process to heat losses and, at the highest temperatures, to internal radiative transfer. Ordinarily, a loss of only 5% of the transferred energy to the surroundings is acceptable, but in this device such a loss would cut the temperature rise in half. With such analytical guidance, the heat losses and radiative transfer were sufficiently reduced by redesign to result in a satisfactory performance.

Again, a particular motivation and various discrepancies between predicted and measured values led to improved performance and better understanding. These finding have ramifications far beyond this application, for example, for the combustion of low-heating-value gases and for the reduction to N_2 of the N_2O now exhausted from medical and dental operating rooms.

Moisture Migration During Conductive Cooling

As a consultant, I predicted the rate of heat leakage from wet soil to well-insulated underground storage tanks for liquified natural gas using the well-known solutions of Stefan and others for conduction with a moving freezing-front. Because of lingering doubts as to the validity of the idealizations inherent in these predictions, I assigned their individual evaluation as research papers in a graduate seminar on heat transfer to Andrew S. Tellar, Li C. Tien, Warren D. Seider, and Lawrence B. Evans. The most surprising and important result was the discovery of the importance of accounting for the heat capacity of the insulation, a quantity that had been almost universally neglected in all of the classical solutions. (See, for example, Seider and Churchill, 1965). One other neglected effect, which, however, required experimental rather than analytical evaluation, was the depression of the freezing point of the water in the capillaries of the soil. Jai P. Gupta undertook to investigate the effect of that phenomenon by measuring the rate of heat transfer to a subcooled metallic surface in a box of wet sand. His measured rates of heat transfer did not differ greatly from my earlier predictions, but his measurements of the local transient concentration of water in the sand were a great surprise. Within the frozen region, the concentration of water greatly exceeded the initial value, implying a rate of water migration more than 1,000 times that predicted for gaseous diffusion of water due to its gradient in partial pressure. We eventually postulated and confirmed that this phenomenal rate was primarily associated with the variation of the surface tension of the liquid water with temperature. These results have many implications beyond the application that inspired the work. For example, they help to explain the accumulation of water under exposed road surfaces in the winter, which leads to potholes in the spring, and they also suggest a means of speeding up the removal of solvents from a porous material, as well as drying in general. Again, an unexpected experimental observation proved to be more important than the primary information that was sought (Gupta and Churchill, 1971).

Thermoacoustic Convection

During their ''spare time'' on the Apollo missions to the moon, the astronauts were asked to carry out simple experiments to confirm the hypothesis of negligible buoyant motion in ''zero'' gravity. The National Aeronautics and Space Agency (NASA) hoped that the absence of buoyant motion would allow the manufacture of better crystals and the achievement of better separations of biological materials in space vehicles than were thought possible on earth. Significant fluid-mechanical effects were, however, observed, and I was one of a number of consultants called in to explain this disappointing result. One initial conjecture was that in the absence of gravity, thermally generated pressure waves might account for the observed fluid motion. A few ''quick and dirty'' experiments revealed that, although such waves were detectable, they were too small in amplitude to explain the observed behavior. We then persuaded NASA personnel to estimate the gravitational vector from the gyroscopic records of the spacecraft. From these results, it was apparent that the net gravitational force during the Apollo flights was not only significant in magnitude, but was also highly variable in direction and time, thereby accounting for the significant buoyant motions. Although we persuaded NASA to use the term ''microgravity'' instead of ''zero gravity'' in the future, and identified the effects of ''g-jitter'', that agency has not yet abandoned all hope of ''manufacturing in space.''

I remained intrigued by the concept of thermally induced pressure waves and sought unsuccessfully for support for research on this topic from NASA. Eventually, I did attain support from the National Science Foundation, in part because of the possible relevance of this phenomenon to the ''Star Wars'' effort. The first quantitative experimental measurements of such waves by Matthew A. Brown were a great disappointment, because they were not in accord in some details, such as wave shape, with the several prior finite-difference and idealized analytical solutions, including our own, all of which were in reasonable agreement with one another. Accordingly, he greatly refined the experimental setup and measurements and achieved an order-of-magnitude improvement in precision, only to find better agreement with his prior crude measurements rather than with the various theoretical predictions. He then turned his attention back to the various numerical solutions, all of which showed some signs of instability despite the apparent convergence of the amplitude of the pressure wave with decreasing grid size. It proved necessary to reduce both the grid size and the time-step by factors of 10^4 to eliminate these small oscillations. The refined numerical predictions then agreed closely with his measurements in every detail. The apparent convergence of the prior numerical solutions proved to be an unfortuitous consequence of the choice of an insensitive variable as a criterion. Strange as it may seem, an error was soon thereafter discovered in one of the early analytical solutions, whose correction also brought its predictions into functional accord with the measurements of Brown. One of the surprises of the final results is the determination of a discrete threshold for an exponential rate of heating of the boundary surface, below which the thermoacoustic effects are negligible. It may be noted that lightning and thunder are a manifestation of this phenomenon in irregular cylindrical coordinates (Brown and Churchill, 1999).

Finally, the most important intrinsic result of the investigation of Brown was the discovery that the well-known failure of the Fourier equation for thermal conduction at short times is simply a consequence of its neglect of compressibility. Compressibility, which is finite for all liquids and solids as well as for gases, couples the equation for the conservation of energy with those for the conservation of mass and momentum, and in so doing prevents an infinite rate of heat transfer even for a discrete and instantaneous step in the bounding temperature. A corollary is that the so-called ''hyperbolic equation of conduction,'' which has often been proposed to correct for this failure of the Fourier equation at short times, is a totally invalid concept in that it implies incompressibility while at the same time artificially incorporating a ''sonic'' wave velocity, which is necessarily a consequence of compressibility. This discovery has not yet halted the derivation and publication of ''theoretical'' solutions based on that false model.

Summary and Conclusions

As illustrated by the series of examples for natural convection, progress in the prediction of heat transfer over the past half century bears almost a one-to-one correspondence with the development of computer hardware and software. However, it is also apparent that the concurrent development of effective and efficient special-purpose algorithms for numerical integration by individual investigators has been as essential ingredient. The development of computer-based methods of display for the velocity and temperature fields, again primarily by individual investigators, has made an important supplementary contribution to qualitative understanding.

Forced and natural convection in laminar flow, including film condensation and boiling, thermal conduction, regeneration and, to a lesser extent, thermal radiation, are now generally predictable from first principles using numerical methods although their coupling with other processes and their occurrence in odd geometries may still provide a challenge. The results obtained by *direct numerical simulations*, although currently limited to a very restricted range of conditions, suggest that the general prediction of turbulent convection from first principles is on the horizon. The model-free formulations developed for turbulent flow and convection by the present author and his associates are proposed as a stopgap measure until that day.

Because of the above-mentioned development of computer-based predictive methods, the role of experimentation has changed significantly. Despite great improvements in instrumentation, it is no longer the method of choice for the determination of the dependence on known primary variables over a wide range of conditions. However, experimentation remains essential to test theoretical predictions and to characterize behavior that is not yet modeled with confidence, as for example, with turbulent convection. In the first of these roles the objective should be the greatest possible precision rather than a great body of measurements. Anomalous observations and, hence, discoveries of new effects are more apt to arise from experiments than from computations since the source of such effects is, by definition, unlikely to be included in the model.

Closed-form analysis has not contributed significantly to progress in heat transfer over the past half century, and its role has now been reduced on the mean to providing asymptotic solutions for the construction of correlating equations or highly idealized solutions for testing the consistency of experimental and computed values.

A silent revolution is occurring in correlation. Log-log plots and fixed-power equations are vanishing in favor of theoretically structured algebraic expressions. The greater complexity of the latter is not an impediment with even hand-held calculators. With this transformation, most of the scatter has vanished along with the need for statistical analyses.

Discovery, innovation and progress are generally the result of anomalous observations. Such inconsistencies are in turn generally the consequence of the synergetic interaction of computation, experimentation, and analysis in the exploration of behavior outside the mainstream of heat transfer and even chemical engineering, for example, as described herein, from space missions, nuclear reactors, ''Star Wars,'' and incinerators.

I have no crystal ball with which to predict the discoveries of the next half century or even the next decade. However, I am sure that progress in heat transfer and all related technologies will be more rapid if we are open-minded and receptive to change in even our most treasured concepts and methodologies, if we are alert to developments outside our own field, if we do not abandon experimental work, and, most of all, if we do not neglect exploratory research because its results are unpredictable and possibly of only long-term interest.

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